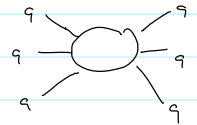


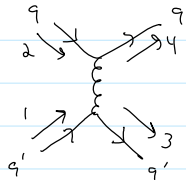
## Example Baryons. $(q_1 q_2 q_3)$

To show that the force between the three quarks in a baryon is attractive when they form a color singlet (and repulsive otherwise) is considerably more tedious.

You might think to evaluate  , but this is complicated and a bit misleading.

The problem is that we might get some "overall" attraction but with a repulsion between two of the quarks.

To analyze this it is easier to first consider pairs of quarks:



$$\Rightarrow M = -\frac{g_s^2}{4} \frac{1}{q^2} [\bar{u}(3)\gamma^\mu u(1)] [\bar{u}(4)\gamma_\mu u(2)] (c_3^\dagger \lambda^a c_1) (c_4^\dagger \lambda^a c_2)$$

Same as repulsive electron-muon scattering.

To evaluate the color factor we need to consider combinations of color assignments for  $q_i q_j'$ .

For  $c\bar{c}'$  we had  $3 \otimes \bar{3} = \overset{\text{octet}}{8} \oplus \underset{\text{singlet}}{1}$

For  $cc'$  we get  $3 \otimes 3 = \overset{\text{triplet}}{3} \oplus \underset{\text{sextet}}{6}$  where  $\left\{ \frac{1}{\sqrt{2}}(rb-br), \frac{1}{\sqrt{2}}(qr-rq), \frac{1}{\sqrt{2}}(bg-gb) \right\} = \text{triplet}$  (anti-symmetric under  $c \leftrightarrow c'$ )

Compare w/ spin- $\frac{1}{2}$  in 3D

$$2 \otimes 2 = 1 \oplus 3$$

$\frac{1}{2}$   $\frac{1}{2}$  scalar vector

$$\left\{ rr, bb, qq, \frac{1}{\sqrt{2}}(rb+br), \frac{1}{\sqrt{2}}(qr+rq), \frac{1}{\sqrt{2}}(gb+bg) \right\} = \text{sextet}$$

(symmetric under  $c \leftrightarrow c'$ )

The color factors for any combination of the two types are:

$$f_{\text{trip}} = -\frac{1}{3} \Rightarrow \text{attractive}$$

$$f_{\text{sex}} = \frac{1}{3} \Rightarrow \text{repulsive}$$

totally symmetric decuplet, e.g.  $\frac{1}{\sqrt{6}}(rgb+bgr+rbg+brg+qrb+qbr)$   
rrr, etc.

Now if we combine 3 quarks we have:  $3 \otimes 3 \otimes 3 = 10 \oplus \overset{\text{mixed symmetry}}{8} \oplus \overset{\text{octets}}{8} \oplus 1$

totally antisymmetric singlet  
e.g.  $\frac{1}{\sqrt{6}}(rgb-rbg-qrb+brg+qbr-bqr)$   
etc.

Only in the totally anti-symmetric singlet configuration is the force between any two pairs of quarks in the baryon attractive. Therefore to reliably predict bound states we need the three quarks in a hadron to form a color singlet.

So the moral of this is that fully attractive interactions which seem reasonable for bound states, require colorless singlet configurations of quarks. This is not a rigorous proof.

On the other hand, the  $SU(3)$  symmetry only allows colorful octet gluons.

There are two experimental observations that we should consider: a) we never see free quarks  
b) we never see free gluons

Both of these are "explained" by the "color confinement hypothesis", i.e. that any free (or long lived) state in QCD must be in a color singlet configuration. This rules out gluons and single quarks, but obviously allows mesons and baryons!

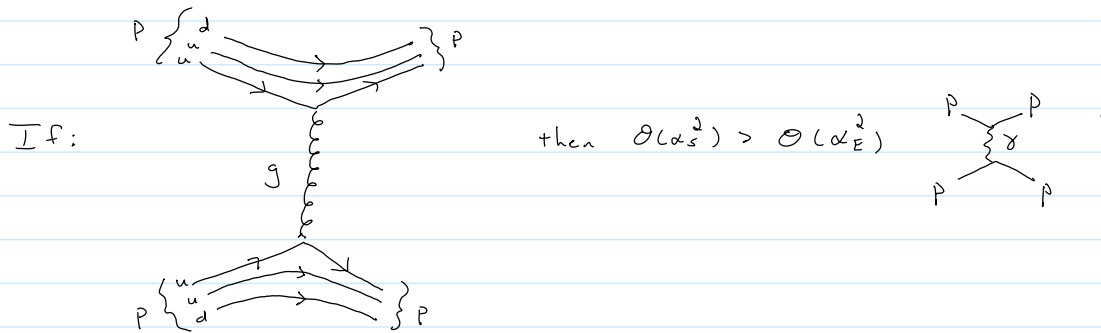
Now proving this hypothesis is one of the great outstanding problems in particle physics. For reasons we will see when we study renormalization, it has a lot to do with the breakdown of perturbation theory (the QCD coupling  $g > 1$ ) in certain situations, so the Feynman approach is useless. Exact methods like lattice gauge theory, supersymmetry and dualities might eventually shed some light.

For now, let's take this hypothesis as a given and look at some implications.

## Strong Interactions Sec 4 Weak

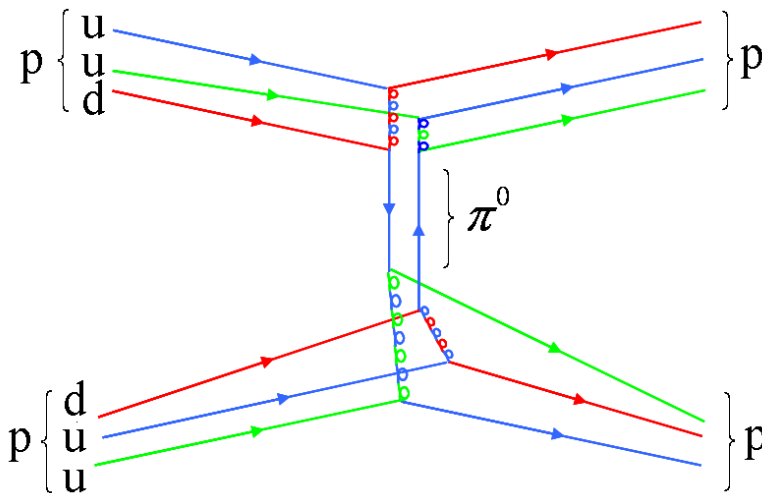
Even though  $\alpha_s > \alpha_w > \alpha_E$  and the gluons are massless, QCD manifests itself as a relatively weak force (compared to EM) between two well separated protons.

Consider proton-proton scattering  $p + p \rightarrow p + p$ :



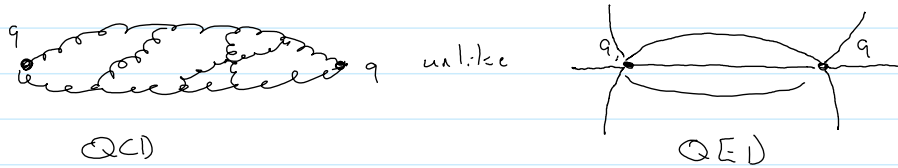
But there are two problems here. First of all each proton is an overall color singlet, so the QCD interaction is a bit more akin to the Van der Waals interaction between two neutral atoms. Additionally, since they are "well separated" the mediator is reasonably long lived and thus should be a color singlet (by confinement), so it cannot be a gluon.

Okay, so how do 2 protons interact via QCD? Consider one possibility:



- There are several important observations.
- Overall this looks like the two protons exchange a pion, which, as a meson, can happily exist as a color singlet.
  - Compared to the QED process which is  $\mathcal{O}(\alpha_E^2)$  this is  $\mathcal{O}(\alpha_s^2)$ . So this will usually be swamped by the EM version.
  - The attraction between  $p$  &  $n$  has an almost identical diagram, but in that case no electromagnetic competition.
  - This entire picture is simplified, in part because each proton is really in  $\frac{1}{\sqrt{6}}(rgb - rbg - grb + brg + gbr - bgr)$  while the  $\pi^0$  is in  $\frac{1}{\sqrt{3}}(r\bar{r} + b\bar{b} + g\bar{g})$ .

An early strategy (prior to QCD) to explain the absence of free quarks was based on the idea that the force between separated quarks gets confined to a tube connecting them. These color flux tubes may very well occur in QCD in part due to the gluon-gluon attraction:



⇓



↑ Studying the dynamics of these fluxtubes ( $\sim 10^{-15}$  m) led to problems like tachyons, spin-2 excitations, etc.

Eventually a couple of folks said that if we shrank these to  $10^{-35}$  m, then the spin-2 state could be the graviton, and by adding supersymmetry we could get rid of the tachyon. Thus was born String Theory.